Linear-quadratic Regulator (LQR) Controlling the Operation of the DC Motor in a Hybrid Wheeled Vehicle

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Abstract

The paper describes the concept of a wheeled vehicle control system with a hybrid powertrain in SiL (Software in the Loop) technology using the NI LabVIEW software. The modeled control system has been constructed based on a mathematical model of a wheeled vehicle and the mathematical models of powertrains. The control of powertrains is provided with type PI controllers for the internal combustion engine and the linear-quadratic regulator (LQR) for the DC motor.

Keywords

LQR Regulator; Hybrid Powertrain; NI LabVIEW

Introduction

The control of a hybrid wheeled vehicle is very hard to carry out in practice. Many aspects associated with the vehicle motion (in particular external forces applied to the wheeled vehicle – resistance to motion), must be taken into account in the control algorithm.

A wheeled vehicle powered by an IC engine/electric motor provides much better dynamic properties and features better efficiency than any conventional solutions. In addition, an Internal Combustion (IC) engine working under higher loads is more eco-friendly (emits much less toxic substances to the environment). On the other hand, the electric motor does not require a clutch, while the maximum torque is available already at minimum RPM values, thanks to this, it can be used to accelerate a wheeled vehicle in the initial motion stage.

Hybrid powertrains can be divided into 3 groups [1,2,3]:

- serial structured (Fig. 1),
- parallel structures (Fig. 2),
- power synergy based (Fig. 3).

The serial structured ones are used in vehicles with an electric motor being the main power source. The electric motor is used to accelerate and drive the vehicle, while the IC engine drives the electric generator that provides sufficient amount of energy to power the electric-traction motor. In this case the IC engine usually is operated within an optimal RPM range, i.e. within such a range where the power and torque values are optimized with the need for the electric energy in mind.

On the other hand, the parallel structure makes it possible to transmit torque to the wheels of the vehicle both from the electric and IC unit. The IC engine in this situation can drive the wheeled vehicle or its electric generator.

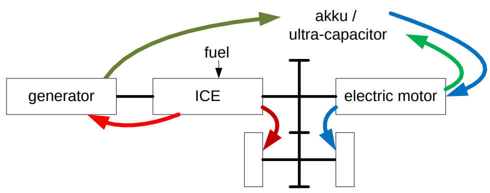

FIG. 1 SERIAL STRUCTURE OF A HYBRID POWERTRAIN [1]

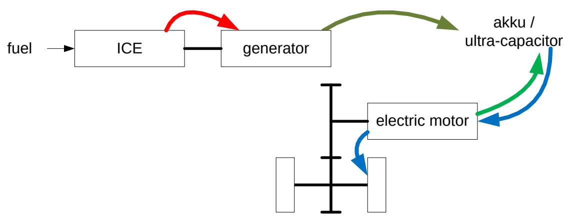

FIG. 2 PARALLEL STRUCTURE OF A HYBRID POWERTRAIN [1]

The energy synergy is mainly used in wheeled vehicles, where power from both drive units is used - hybrid cars, e.g. Toyota Prius.

The choice of proper structure for combining power

units depends mainly on the wheeled vehicle application (traffic in urban environments - a small passenger vehicle, intercity traffic, interstate traffic – huge carriage trucks) and the very costs of manufacturing a complex powertrain.

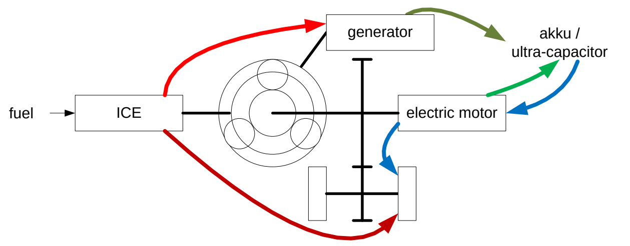

FIG. 3 HYBRID POWERTRAIN USING ENERGY OF SYNERGY[1]

Theoretical Solution for the Electric Motor

The simulation of a hybrid powertrain has been performed using the LabVIEW software supplied by National Instruments, where a mathematical model for the DC motor has been assumed (DC Motor, Fig. 4). The simplest description of the DC motor is provided by the following mathematical equations:

- T torque of the DC motor depends on the current flow in i armature winding and electric machine constant K_T:

$$T = K_T \cdot i$$

counter-electromotive force (back EMF) e describes the change of angular position for rotor $\frac{d\theta}{dt}$ (angular speed) and electric machine constant K_e:

$$e = K_e \cdot \frac{d\theta}{dt}$$

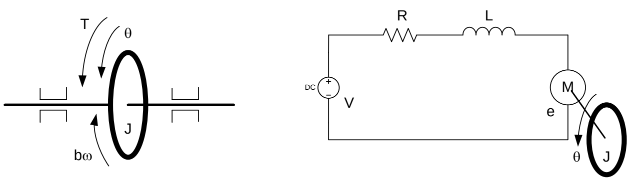

FIG. 4 MODEL OF A DC MOTOR[1]

By using Fig. 4 it is possible to write an equation based on Newton's and Kirchhoff's laws:

$$J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = K_T \cdot i$$

$$L\frac{di}{dt} + R \cdot i = V - K_e \cdot \frac{d\theta}{dt}$$

where:

J – moment of inertia $[kg \cdot m^2]$,

b – mechanical damping$[\frac{Nm \cdot s}{rad}]$

L - inductance $[H]$,

R - resistance $[\Omega]$,

V - voltage $[V]$,

$K_T \left[\frac{Nm}{A}\right] = K_e \left[\frac{V \cdot s}{rad}\right]$.

The DC motor model can be also written in a matrix form used to build state equations. The mathematical description using the state equations is necessary to build the linear-quadratic regulator (LQR):

$$\dot{x} = Ax + Bu => \begin{bmatrix} \frac{di}{dt} \\ \frac{d^2\theta}{dt^2} \end{bmatrix} =$$

$$\begin{bmatrix} -\frac{R}{L} & -\frac{K}{L} \\ \frac{K}{J} & -\frac{B}{J} \end{bmatrix} \cdot \begin{bmatrix} i \\ \frac{d\theta}{dt} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot [V]$$

$$y = Cx + Du => y = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} i \\ \frac{d\theta}{dt} \end{bmatrix} + [0] \cdot [V]$$

The mathematical model of the DC motor in state space presented in Fig. 5 has been prepared by using a transfer function. The DC motor model after transformation from the transfer function to the state space is described with state equations presented in Fig.6. The DC model described with state equations is used to provide a mathematical description of the control object for the linear-quadratic regulator (LQR).

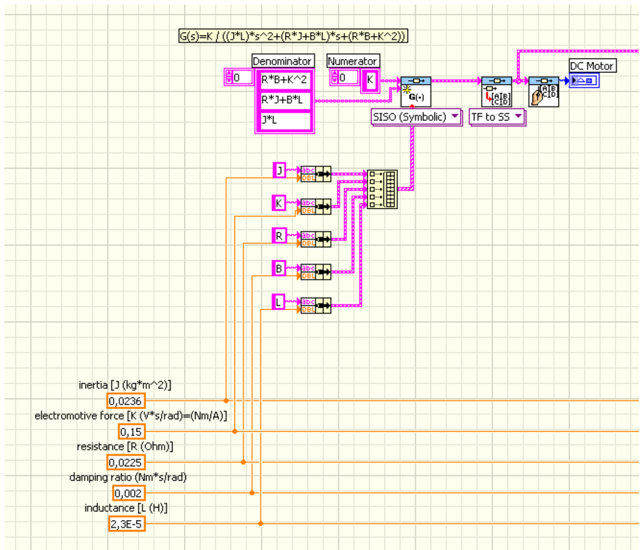

FIG. 5 DC MOTOR DESCRIBED WITH A TRANSFER FUNCTION

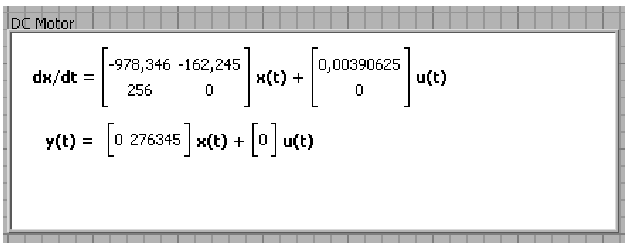

FIG. 6 DC MOTOR MODEL IN THE FORM OF STATE EQUATIONS

The DC motor model has been built by using a commercially available electric motor manufactured by L.M.C. 200, model 127, with the following specifications:

- no load current $(A) = 5\ A$,
- torque constant $\left(K_T\left[\frac{Nm}{A}\right] = K_\theta\left[\frac{V\cdot s}{rad}\right]\right) = 5\ \frac{Nm}{A}$,
- armature resistance $(m\Omega) = 22{,}5\ m\Omega$,
- armature inductance $(\mu H) = 23\ \mu H$,
- armature inertia $(kg\cdot m^2) = 0{,}0236\ kg\cdot m^2$,
- peak current $(A) = 400\ A$,
- rated current $(A) = 215\ A$,
- rated speed $(RPM) = 2600\ RPM$,
- rated voltage $(V) = 48\ V$,
- rated torque $(Nm) = 31{,}5\ Nm$.

The motors manufactured by this company are frequently used in electric or hybrid vehicle powertrains.

LQR in NI LabVIEW Software

The LQR controller provides a solution for the case where the object controlled has been described with a system of differential linear equations in the state space, while the so-called cost (minimized by following the optimum control principle) is described with a quadratic form. In the LQR control the process control is provided by using a mathematical algorithm through minimizing the cost function with assumed function parameters being function weights. While constructing the linear-quadratic regulator (LQR) it is good to present the following basic quality indicators:

$J = \int_{t_0}^{\infty} \varepsilon_p^2(t)dt$ - control error,

$J = \int_{t_0}^{t_1} u^2(t)dt$ - cost of control energy,

$J = \int_{t_0}^{t_1} |u(t)|dt$ - amount of energy expenditure,

$J = \int_{t_0}^{\infty} [x^T(t)\cdot Qx(t) + u^T(t)\cdot Ru(t)]dt$ - the so-called compromise between the control quality (stabilization) and the costs of control determined by Q and R weight matrices.

Quality index

$J = \int_{t_0}^{\infty} [x^T(t)\cdot Qx(t) + u^T(t)\cdot Ru(t)]dt$ presents the problem of linear regulator with a quadratic quality index, with its control features limited in terms of energy, but unlimited in terms of amplitude. As a result, the optimum control can be described by the following equation:

$$u(t) = -Kx(t) = -R^{-1}\cdot B^T\cdot Kx(t)$$

where the matrix of controller amplification coefficients $K(t)$ is rectangular, which can be expressed in the following way:

$$u_1 = -(k_{11}x_1 + k_{12}x_2 + k_{13}x_3)$$
$$u_2 = -(k_{21}x_1 + k_{22}x_2 + k_{23}x_3)$$

or in the form of matrices

$$\begin{bmatrix}u_1\\u_2\end{bmatrix} = -\begin{bmatrix}k_{11} & k_{12} & k_{13}\\k_{21} & k_{22} & k_{23}\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},$$

which finally adopts the following form

$$u = -Kx$$

Matrix K provides the solution for the algebraic Riccati equation:

$$0 = KA = A^TK - KBR^{-1}B^TK + Q$$

The representation of the LQR controller in LabVIEW software is shown in Fig. 7. One can see the LQR controller and Q and R matrices forming an analytic regulator. The amplification calculated during the analysis enters the regulator control part with some control signal error.

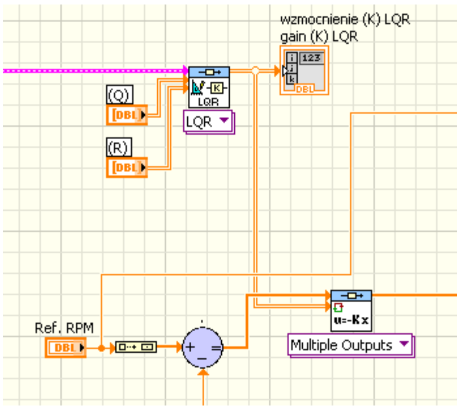

FIG. 7 LQR CONTROLLER PRESENTED IN NI LABVIEW

The following weights for Q and R matrices have been assumed:

$$Q = \begin{bmatrix}100 & 0\\0 & 1\end{bmatrix}$$
$$R = [0{,}01]$$

The control signal corresponds to the difference between the input and measured speed and makes the basis for controller amplification coefficient states $u = -Kx$.

The DC motor (Fig. 8) has been designed by using dynamic equations presented in section 2.

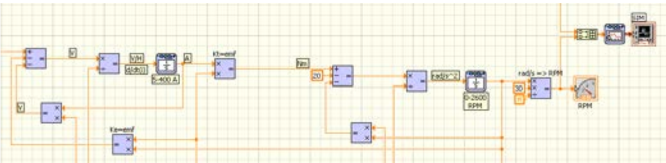

FIG. 8 DC MOTOR MODEL IN NI LABVIEW

The results for the modeled DC motor with the linear-quadratic regulator (LQR) are presented in Fig. 9 and Fig. 10. In the modeled system (Fig. 9): DC motor + linear-quadratic regulator (LQR) there is no control signal distortion, the output signal from the object keeps pace with the input signal, and no oscillation is observed. The measured speed - real (in red) corresponds to reference speed - input speed (in white).

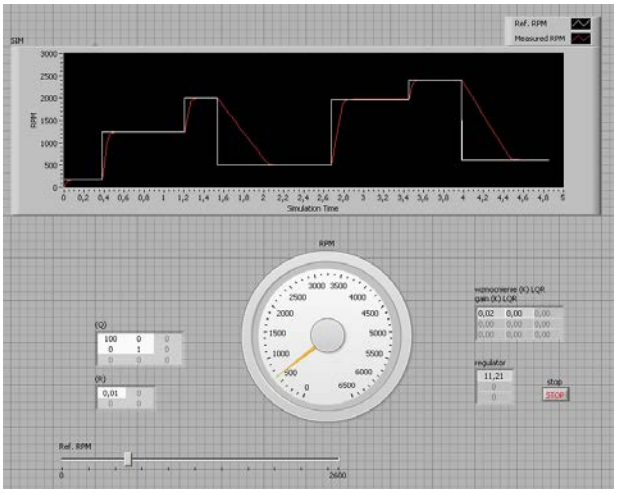

FIG. 9 SIMULATION RESULTS - SPEED

Fig. 10 presents the results of current and voltage tests obtained for the modeled motor. Increasing the RPM results in energy consumption rise (white curve) which represents external resistance torque limited to the motor shaft after the motors reaches the reference speed. Decreasing the revs results in lower motor energy consumption.

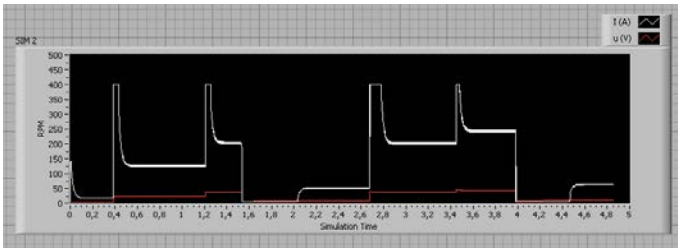

FIG. 10 SIMULATION RESULTS – CURRENT AND VOLTAGE VALUES

The DC motor modeled with LQR controller was used for tests as the drive unit of a hybrid wheeled vehicle. The simulation performed adopted a DC motor + linear-quadratic regulator (LQR) for a wheeled vehicle. The DC motor drives a wheeled vehicle up to a speed of 60km/h, when drive units are switched over and the vehicle is driven by its IC engine. A simulation was performed for the engine tested (Fig. 11), and consisted in accelerating the wheeled vehicle without exceeding 60km/h, then the vehicle was decelerated by inertia (without using brakes). In the second run it was decelerated by using brakes. Once the vehicle stopped, it was re-accelerated up to a speed exceeding 60km/h.

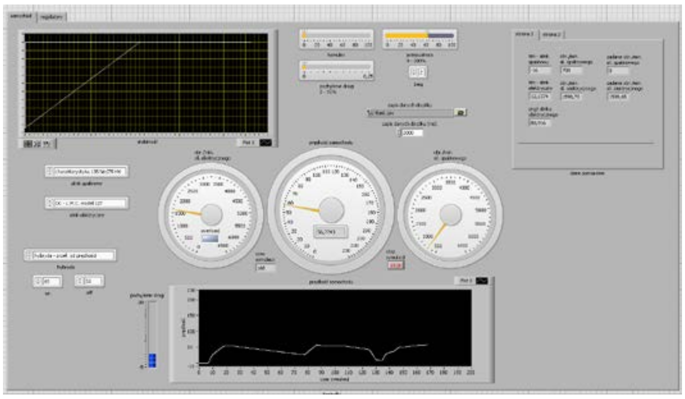

FIG. 11 SIMULATION RESULTS PERFORMED FOR A HYBRID WHEELED VEHICLE

Conclusions

The paper presents the way of modeling a DC motor with a linear-quadratic regulator (LQR) being a part of hybrid wheeled vehicle. A mathematical model of the DC motor in the form of differential equations, transfer functions and state equations have been presented. The mathematical description of the DC motor in the form of state equations was used to build a virtual linear-quadratic regulator (LQR). A simulation of a powertrain using the DC motor operating with linear-quadratic regulator (LQR) has also been performed. The simulation performed confirms the possibility to use the linear-quadratic regulator (LQR) to control wheeled vehicle driving motor both in electric and hybrid powertrains. Adopting this solution will help to improve the powertrain dynamic properties and, thorough eliminating oscillating control signal, to enhance comfort.

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Gabriel Kost (was born 1960) in 1984 graduated from the Silesian University of Technology in the Faculty of Mechanical Engineering, and he got a degree of mechanical engineer in speciality of machine technology and began work at the Institute of the Machine Building. In 1991 he was given a doctor's degree of technical science, and in 2005 a doctor of science degree in the scope of the robotization of technological processes. He is interested in problems of automation and the robotization of technological processes, of-line programming and motion planning of industrial robots.

Andrzej Nierychlok (born in 1980) graduated from the Faculty of Mechanical Engineering in 2008. In the same year he began PhD studies. He is interested in processes automation including transport equipment and industrial robots, CNC control, and computer software CAD / CAM / CAE. In the years 2005-08 he worked in industry as a process engineer and designer of lifting devices.